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THE MATHEMATICAL FACTS OF GAMES OF CHANCE BETWEEN EXPOSURE, TEACHING, AND CONTRIBUTION TO COGNITIVE THERAPIES: PRINCIPLES OF AN OPTIMAL MATHEMATICAL INTERVENTION FOR RESPONSIBLE GAMBLING

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Abstract

On the question of whether gambling behavior can be changed as result of teaching gamblers the mathematics of gambling, past studies have yielded contradictory results, and a clear conclusion has not yet been drawn. In this paper, I bring some criticisms to the empirical studies that tended to answer no to this hypothesis, regarding the sampling and laboratory testing, and I argue that an optimal mathematical scholastic intervention with the objective of preventing problem gambling is possible, by providing the principles that would optimize the structure and content of the teaching module. Given the ethical aspects of the exposure of mathematical facts behind games of chance, and starting from the slots case – where the parametric design is missing, we have to draw a line between ethical and optional information with respect to the mathematical content provided by a scholastic intervention. Arguing for the role of mathematics in problem-gambling prevention and treatment, interdisciplinary research directions are drawn toward implementing an optimal mathematical module in cognitive therapies.

Cuvinte cheie: *matematica jocurilor de noroc; aparate de sloturi; teoria probabilitatilor; interpretarile probabilitatii; psihologia probabilitatii; etica jocurilor de noroc.*

Keywords*: gambling mathematics; slot machines; probability theory; interpretations of probability; psychology of probability; gambling ethics.*

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1. INTRODUCTION

Problem gambling is one of the fields patronized exclusively by psychology, as it came about naturally as one of the social effects of the gambling phenomenon. Mathematics is strongly connected to gambling through the mathematical models underlying any game of chance. Games of chance are developed structurally and physically around abstract mathematical models, which are their mere essence, and the applications within these mathematical models represent the premises of their functionality (for instance, the house edge is ensured through precise calculations regarding expected value; if such calculations were not possible, the game would never run). Since in problem-gambling research, treatment, and prevention we cannot separate the gambler from the game he plays, it follows that an optimal psychological intervention cannot disregard mathematics. Call this the gambling-math indispensability principle.

Thus far, the mathematics of gambling has been a subject of interest more for gamblers than psychologists, and the plethora of literature on gambling mathematics for the popular audience in the last decade confirms that observation. As regards psychology, the role of mathematics has been limited to *providing* odds (of winning/losing) and statistical indicators, and adjusting erroneous beliefs and fallacies related to probability and randomness. Empirical studies have been conducted *testing* hypotheses related to how gambling behavior changes with this mathematical knowledge, but those studies did not yield conclusive results. The relationship mathematics has developed with psychology in the course of such research is a *indirect* one – the mathematical intervention is addressed exclusively to gamblers via a third-party resource, and psychology only conducted the empirical studies and interpreted the results in terms of predicted behavior. In conclusion, the *direct* contribution of mathematics to psychological intervention in problem gambling was reduced to *facing the odds* and *correcting misconceptions*. However, these interventions are not enough, and some of the past empirical studies have confirmed that statement.

Following the gambling-math indispensability principle, mathematics can go deeper into the gambler's mind with the help of psychology (or conversely) and its contribution can extend further to cognitive therapies, going beyond Probability & Statistics and incorporating knowledge from adjacent domains such as mathematical modeling, decision theory, theory of representation, and even epistemology (Bărboianu, 2013c).

The current paper is focused on the indirect contribution of mathematics to responsible gambling through an optimal scholastic intervention and draws on the further research needed for establishing and implementing a direct contribution of it into the psychological interventions.

2. EXPOSURE OF THE MATHEMATICAL FACTS OF A GAME AS AN ETHICAL OBLIGATION

2.1. The slots case – an unjustified secrecy of their parametric design

The slot games have gained and maintained top popularity despite their non-transparency with respect to parametric configuration, as this information is not exposed. Slots remains the only game in which players are not aware of the essential parameters of the game, such as number of stops of the reels, number of symbols, and their distribution on the reels. Obviously, the lack of data regarding the configuration of a machine prevents people from computing the associated odds of winning as well as other mathematical indicators.

The so-called PAR sheets, exposing few of the parameters of the machines and probabilities associated with the winning combinations, are kept secret by game producers and can be retrieved only upon request via legal action in some jurisdictions–for example, through the Freedom of Information and Protection of Privacy Act, in Canada (Harrigan & Dixon, 2009).

Fortunately, mathematics provides us with statistical methods of retrieving the missing parametric data based on long-run observation, as approximations (refined through methods based on numerical analysis and pattern recognition); however, such methods require considerable effort to put into practice (for instance, recording the outcome of thousands of spins for each reel, done by volunteers) (Bărboianu, 2013a). In fact, the existence and theoretical applicability of these methods of retrieving the missing data are in and of themselves arguments for the insubstantiality of the secrecy of slot producers on their PAR sheets.

Nor do slot producers have a valid justification with respect to the company's interests. In the appeal decisions of the Information and Privacy Commissioner (IPC) in Ontario, Canada, with respect to declined PAR sheets requests, producers who declined the requests invoked the exemption set forth for scientific and technical information, considering that PAR sheets are trade secrets in the gaming industry and their exposure can significantly prejudice the competitive position of the company. (Information and Privacy Commissioner [IPC], 2009, 2010).

The slot producers' reasons, shown in the IPC's appeal decisions, seem to be judicially formal rather than factual because: a) the trade secret and intellectual ownership reasons fail against the generality of the math formulas and equations since although the parametric details vary from game to game, the mathematical results are just *applications* of general formulas that are publicly available in mathematics and common across all slot machines; b) the competitive prejudice reasons fail against the open possibility for all slot producers to configure, test, and use any parametric design for their slot machines, which can be manipulated in unlimited ways, so as to obtain the desired statistical indicators for the house.

Finally, slot producers have no valid justification with respect to their players. The hypothetical reason of being afraid of losing players who face the real odds and expected values of their games fails against the *a priori* expectation of the players for low and very low odds of winning induced by the secrecy of PAR sheets that they have encountered, and against the lottery example, in which lottery players keep playing against the (well-known) lowest odds of winning due to other addictive elements that slots also hold (Bărboianu, 2013b, forthcoming).

2.2. The mathematical facts between ethically required and optional information

The slots case raises the problem of the obligation to expose the *parametric configuration* of any existent or forthcoming game of chance, even though it currently applies only to slots.

The goal of exposing the parametric configuration of the slot games is not necessarily to acquire for slots the same status as other games of chances in this respect, but rather, in the respect of ethics. Exposing the parametric configuration of a game to the player prior to playing is an ethical obligation in two aspects – one commercial and the other humanitarian.

The commercial aspect treats the game as any commercial service, for which full technical specifications are required from the producer to the customer; a bet is still a purchased service once the player inserts a non-returnable coin in the machine.

The humanitarian aspect is related first to the free will of thought and second, to the limitation of the risk factors through further improved knowledge. Being informed on all parameters of a game one plays is a condition for unconstrained personal thinking leading to personal actions. It is as if someone asks you to bet you can jump from a high place and land on your feet; of course, if you know in advance the height from which you will jump, or measure it before you bet, you might decline the bet or propose another one for a certain measurement, and this means *free decision*.

Regarding the limitation of risk factors through further improved knowledge, acquired either as pre-calculated numerical results such as winning odds and other statistical indicators, or by learning theoretical and applied probability theory basics, that is the subject of the next section.

The information required to be exposed as parametric configurations would be in the form of a technical/mathematical sheet specific to each game, consisting of those parameters of the mathematical design of that game that define the sets of possible outcomes and are essential toward probability and statistical computations. For example, in slots the parametric-configuration sheet must show the number of distinct symbols, number of stops of each reel, and the symbol distribution (weighting) of each reel. In a card game, the number of decks used, the number of cards in each deck, and the composition of each deck (numbers of card values and symbols) are known. With a drawing machine (for example, lottery or bingo), the total number of numbers/balls, their value interval, the number of numbers/balls to be drawn, and so on are likewise known.

The ethical obligation being established, the question arises as to whether this obligation should remain simply the parametric configuration of the game or be extended to include basic or advanced mathematical results coming from applications worked out on the mathematical model of that game. The extension could consist of basic pre-calculated numerical results, such as probabilities of the basic winning events and expected value, or stretch further to more complex gaming events and other statistical indicators, and the interpretation of these results. The latter variant already assumes a new level of mathematical knowledge, attainable only through scholastic intervention. For the parametric-configuration-only variant, which is merely informative and either provided by the game producer or retrieved by third parties, it would remain for the player to inquire further for the mathematical results as an optional action.

The question, then, clearly becomes where to draw the line between ethically required and optional information on the mathematical facts of games of chance.

Once the line drawn, the obligation would be imposable only by law, since game operators, like game producers, might consider that it is not to their advantage to provide such technical/mathematical sheets to their customers.

On the entire range of mathematical information possible to be exposed, as the amount of information increases, there are three specific levels as seen in the next figure: parametric configuration, basic numerical results (odds of winning and EV), and knowledge of the mathematics of gambling presented in a specific teaching module. Interval I from the first to the second level does not have intermediary values, while interval II – marked with a continuous line in the figure – could have very many intermediary values, depending on the amount and structure of the exposed mathematical information.

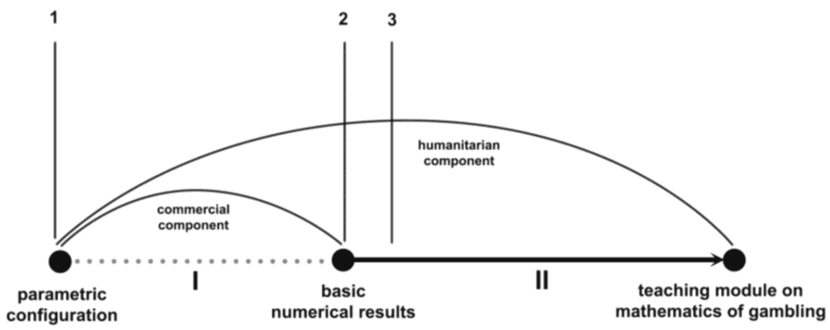

Figure. Dividing the range of the mathematical information for a game of chance between ethically required and optional information.

If assigning the two aspects – commercial and humanitarian – of the ethical obligation over the range of the mathematical information, the commercial one covers only interval I, while the humanitarian aspect could stretch theoretically to the endpoint of interval II, if enhanced mathematical knowledge can have an impact on gambling behavior.

Indeed, we cannot extend the coverage of the commercial component beyond the exposure of the basic numerical results, since imposing this extra effort on game producers would be unethical/unfair, given that they already provided the parametric configuration and the basic results that can be further developed optionally by players with the help of other qualified entities. Such a requirement would be comparable to requiring drug producers to expose on their informational leaflets not only the substances in the composition of the drug and the numerical results of the statistical studies on drug's side effects, but also medical information on the possible side effects, while this information is available optionally through medical consultation. Examples aside, the practice of exposing a product/service's information based on commercial ethics confirms this limitation in any economic/commercial field.

As for the humanitarian component, even though its coverage could be established only through subjective criteria coming from the involved entities, there is also an objective limitation imposed by the extra time and effort that attendance for a course requires from gamblers. In addition, imposing on game producers a requirement to maintain, sustain, or support such courses would also be unethical/unfair, for the same reason presented for the commercial component.

In conclusion, if reducing the ethical obligation to its commercial component, there are two options for drawing the line between ethical and optional information: at *parametric configuration only* for ethics (position 1 in the above figure) or *parametric configuration plus basic numerical results* for ethics (position 2 in the above figure), and a final choice can be made only by legislators. Choosing position 2 would be somehow in the vein of the ethical information exposed on cigarette packs, where not only the substances contained in tobacco smoke are mentioned (equivalent of position 1 in our account), but also the warning on health injuries caused by cigarettes and sometimes statistical data on cancers caused by smoking (equivalent of position 2). Of course, in many respects the two situations are not equivalent.

If keeping the humanitarian component as necessary for defining the ethics in this particular domain, the line would lie in interval II (including position 2), at a position yet to be established through the consultation of the communities involved (gamblers, game producers, problem-gambling scientific communities, and other specialists) before a choice is made by legislators.

Compromise options would still place the line in interval II, in the proximity of position 2. For instance, such an option would be the exposure of the parametric configuration, basic winning odds, expected value, warnings toward gambling fallacies, misconceptions and misinterpretations of the exposed results, and optional recommendations to attend gambling mathematics courses for a better understanding and interpretation of the mathematical facts that govern the game.

At first glance, the *best* option seems to lean toward position 2, which also has the highest number of corresponding examples from other domains; however, further interdisciplinary research is necessary for a rigorous standard, including how "best" should be defined in this particular ethical context.

3. THE OPTIMAL MATHEMATICAL SCHOLASTIC INTERVENTION FOR RESPONSIBLE GAMBLING

It is necessary before proceeding toward an optimal mathematical scholastic intervention in gambling to decide whether such an intervention would accomplish the goal of limiting the risk factors and result in a significant desirable change in gamblers' behavior. Regarding the setting in which such an intervention could take place, there are three non-exclusive options: in secondary to post-secondary public schools as optional course or module attached to the probability/statistics courses, within private companies or institutions dealing with gambling and problem gambling, and within cognitive therapy sessions for pathological gambling, strongly reduced to conclusive knowledge and guidelines implemented by the therapist and applied through psychological counseling. The principles stated in the later section *Principles of an optimal mathematical scholastic intervention to gamblers* apply to the first two of these options.

3.1. Theoretical versus empirical studies on the impact of scholastic intervention

In the literature on this matter, contradictory results have been published and a clear conclusion has not yet been drawn. Most of the results were based on statistical studies of college-student gamblers who received a scholastic intervention; some of these results were declared by their authors as "paradoxical" or "unexpected," as they did not confirm the expectation of a significant change in the gambling behavior of the subjects. Thus, Hertwig et al. (2004) found that students who received education on probability gambled on low-odds events more than the students who did not know the actual odds; Steenbergh et al. (2004) found that students who were taught about and given warning about gambling fallacies and mathematical expectation gained superior knowledge on these matters, but were just as likely to play roulette as students who did not receive this intervention; Williams & Connolly (2006) found that students who received instruction on probability theory applied in gambling demonstrated superior ability to calculate gambling odds, as well as resistance to gambling fallacies, but this enhanced knowledge was not associated with any decreases in actual gambling behavior. On the other side, additional theoretical studies proved that post-secondary statistics education developed critical thinking, which also applied to gambling, and gamblers who get such education tend to have significantly lower rates of problem gambling (Gray & Mill, 1991; Gerstein et al., 1999; Abbot & Volberg, 2000).

I think that the approach to the problem of changing gambling behavior as result of the mathematical scholastic intervention must be more theoretical than empirical, even though it assumes the use of psychological tools of evaluation. The main reason is that a proper testing of the hypotheses or expectations of the empirical studies is only marginally attainable – if not impossible – since gamblers much be monitored over a long time with no constraints on their actions; the monitoring period should take into account each gambler's own frequency of playing and other personal parameters; therefore, a unique overall monitoring period for the entire sample group cannot be determined with respect to the relevance of the results – a very long monitoring period is needed; as for the constraints, participation itself in the study apprises gamblers of the expectations of the study, which from the outset becomes a constraint that might influence his/her actions. For example, given the newly acquired mathematical knowledge, a gambler could be keen to see whether this knowledge can be applied strategically in the games of chance, resulting in profits, and this attitude could result in an initial increase of his/her gambling activity after the intervention – which could also decrease later in the absence of the anticipated results. (Even though the mathematical facts were taught with the goal of limiting gambling activity, such a trial period on gambler's side could result in moving from one type of game to another more suitable for probability-based strategies.) This potential behavior toward the strategic use of mathematics could explain the "unexpected" results of the studies mentioned above, among which the reported increase in the gambling activity was consistent.

Besides the monitoring period, there are also several issues with the conditions of testing the hypotheses of the empirical studies (for instance, the gambling activity being measured in money or time spent, the trustworthiness of the gamblers' reports given the pathological aspect of problem gambling, the quantification in case of playing more than one type of game, etc.). All of the studies mentioned had a laboratory-based evaluation of gambling behavior for testing the hypotheses, which cannot reproduce real-world gambling activity – filling a questionnaire on future intentions can neither substitute for nor predict real actions.

Another criticism of the performed empirical studies concerns the sampling. All mentioned studies were undertaken on groups of *college* students, which is not a representative sample for the gambler population with respect to age. The argument for choosing that category of gamblers was that official reports have found the rate of problem and pathological gambling to peak in ages 18 to 24. My argument for extending the age criterion beyond 24 is twofold: first, the final goal of a scholastic intervention is to limit the risk factors for *all* gamblers, already manifested or potential, given that in time, a non-problem gambler can become a problem one; second, the age interval 18 - 24 assumes a particular psychological profile whose features could affect the outcome of the intervention. It is well known that young persons – although more open to learning than older persons– are interested in filling their spare time with entertaining activities more so than their elders, and gambling seems to be one such activity. This status of their gambling activity could prevail over the other main reasons for gambling that older persons may have, of which winning money is the most important. In addition, older gamblers have experienced the gambling failure (money and time spent versus profits gained) more than the younger ones and scholastic intervention could find a more favorable ground in the age range over 24. For these reasons, I expect to see different results on the impact of the mathematical scholastic intervention from empirical studies using a representative sample of the gambling population with respect to the age criterion.

One can object that a mathematical background is essential for application of the intervention, and college students are the most likely to have such a background. I answer that in case of a non-math gambler, the intervention can be reduced to the simple delivery of numerical odds and statistical indicators, along with a basic interpretation of them (this is actually the practical side and main goal of any advanced learning) and the studies can test the same hypotheses under this condition.

I also claim that the sample should be representative for the locale, as different economic environments can affect the intensity of gambling activity where money is involved, of course if such studies have an international focus and sampling.

The structure and content of the teaching module is also important toward the effects of the intervention, and we cannot draw a complete conclusion on the similarity of the reported empirical results if the teaching module for each intervention has a different structure. The structure of the teaching modules is also the subject of the next section on the optimal mathematical scholastic intervention.

In conclusion, further theoretical interdisciplinary research is needed on the impact of the mathematical intervention on gambling behavior and also on the optimal conducting of the statistical studies on representative samples from the gambler population; these enhanced empirical studies could confirm the theoretical results. I am inclined to think that a decrease in the gambling behavior can be the result of an optimal intervention (despite the reported results of past statistical studies).

Overall, to the question of whether a mathematical scholastic intervention is worth studying, developing, and putting into practice, I answer positively. Even considering as nonconclusive the studies to date on the impact of such an intervention, the intervention falls partially within the ethical obligation to expose the mathematical facts behind games of chance, and an optimal exposure assumes not only numbers, but also interpretations and warnings, which have a scholastic component.

3.2. Principles of an optimal mathematical scholastic intervention to gamblers

Although over the last two decades, probability and statistics were present in the curricula of most of the secondary schools as well as some 7th and 8th grades worldwide, studies have indicated a decrease in the role of probability and a greater focus on data processing at these educational levels (Borovcnik, 2006). Among the reasons given for this decrease (one of which is that probability is oriented too much toward advanced mathematics, which makes it a difficult topic to teach at the secondary school level), there is the puritanistic view that probability is too closely connected to games of chance, which are seen as plagues of contemporaneous society even in the jurisdictions where they are legal (Borovcnik, 2006). Of course, this principle directly affects the structure of the respective teaching modules, unfortunately lacking sufficient examples and applications from the games of chance, which are essential for a good understanding of probability theory. With this trend, probability theory came to be taught in schools only because it is necessary to justify the methods of inferential statistics. Besides the contradiction with the genesis of probability theory and the concept of probability itself (which were born in the 17th century from games of chance), and with the optimality of the learning process that according to my view, knowledge of the mathematics of gambling can have an impact on gambling behavior if properly taught, that puritanistic principle becomes paradoxical: teaching is modified to avoid mentioning games of chance as much as possible, while on the contrary, understanding the mathematical facts of these games can have a decisive role in limiting the risk factors. However, this principle is not applied in all countries. The best example is Australia, where not only do syllabi outline the role of probability in everyday life and decision making, but teaching modules on the mathematics of gambling have been implemented with success in the secondary schools. In 2008, in the state of Queensland, mathematician Robert Peard developed and helped to implement through governmental intervention a teaching unit called *The Mathematics of Responsible Gambling* (Peard, 2008).

The current research is not focused on probability and statistics courses from the school curricula with respect to problem gambling (which still can remain a good background for further learning), but on developing an optimal teaching module on the mathematics of gambling which will be applicable for both potential and experienced gamblers, with the goal of limiting gambling risk factors and controlling gambling behavior against its pathological side. Such a teaching module would remain optional for the gamblers, as it cannot be imposed through regulations, and offered in both governmental and private venues. However, the possible imposition by law of the exposure of the parametric configuration plus the basic numerical results (such as probabilities of the main winning events and expected values) as an ethical obligation for all games of chance might encourage gamblers to attend such a teaching module on their own for a better understanding and use of the exposed results.

When defining the optimality of the mathematical scholastic intervention to gamblers through principles, we should relate them to the main goals of the intervention, which are:

1. All gamblers should be able to attend the teaching module and understand the basic knowledge taught, regardless of their level of mathematical education.[1]
2. The gambler will understand the nature and interpretations of the probability concept, its relativity toward the practical aspects of its use in making decisions, and the relation between the probabilistic models and the real world; and he/she will have a clear image of the concepts of randomness and independence.
3. The gambler will be able to perform basic probability computations, evaluations, and approximations for the various gaming events encountered, expected values, and to search for pre-calculated results from available resources.
4. The gambler will show resistance to all gambling fallacies specific to any game.
5. The gambler will evaluate mathematically his gambling activity for short and long term; he/she will finally have in mind an abstract representation of the games he/she plays by reducing them to their mathematical models, and thereby ignoring their addictive elements added in the physical state.

These goals can be accomplished mainly through (but not limited to) the structure and content of the teaching module. The following principles are important for an optimal structure and content, and the purpose of this paper is just to claim them as decisive toward the proposed aim without generating the entire detailed structure of the module. This structure and the inference on why and how these principles can induce the sought-after effect on gambler's behavior will be the matter of a forthcoming interdisciplinary research and scholastic project. These principles are stated below:

a) The teaching module must be adapted to all levels of background mathematical education, which will be established through preliminary tests. For the lower levels, the module will be extended with additional preliminary lessons as the level requires, having topics such as real numbers, numeric calculus, functions,

[1] A degree of a secondary school is required to attend this teaching module, so the use of the terms "all gamblers" and "gamblers" assumes this prerequisite.

algebraic calculus, and set theory basics. Also for the lower levels, the lessons within the probability and statistics parts will be enhanced with more examples and a more extensive interactive component. Some lessons will be split and completed with extra examples. This approach requires teaching a large part of the module in separate groups of different levels and reduces the risk of rejection and abandonment of the intervention by the gambler for reasons of incomprehension and inadaptability. This principle is related to goal 1.

b) The theoretical parts should be limited in generality strictly to cover through application the games of chance, which means teaching only in discrete probability and only those results facilitating understanding of the basic concepts and the computational purpose. Exceptions are applicable if serving the purpose of clarifying a concept (for instance, probability as a measure requires a good deal of generality). Any added advanced mathematics not serving the purposes of the intervention could result in a break in the student's connection with the teaching flow. This principle is related to goals 1, 3, 4, and 5.

c) The module should have a strong applicative character, showing the student how to frame each game and gambling problem within the suitable probabilistic model to which theory is applied, and conversely, each theoretical asset should be followed by solved applications from gambling. For the student to acquire computational skills, an algorithmic approach of the applications is required. This principle is related to goals 1, 3, and 5.

d) The module should have compact sub-modules, each dedicated to one major game of chance, where the most important applications specific to that game are presented. Such sub-modules should also have collections of pre-calculated numerical results for the student to study and assign to imaginary gaming situations, thereby facing as many probabilities as possible. Students who practice only one game may attend only the sub-module dedicated to that game. This principle is related to goals 1, 2, 3, 4, and 5.

e) The module should have a compact sub-module dedicated to gambling fallacies, misconceptions and erroneous interpretations of theoretical and numerical results from probability and statistics, even though these subjects are touched upon in other theoretical lessons. This principle is related to goals 2 and 4.

f) Applicative lessons and seminars should contain recommendations and instructions on the choosing and use of external resources on gambling mathematics, given the wide exposure on the internet and the large number of book titles on this topic. This principle is related to goal 3.

g) The applications toward strategy and optimal play, presented in a sub-module dedicated to a specific game, will be limited and focused on winning odds and the long-run play of that game. The student will be taught that an optimal play would give him/her advantage in a game against opponents, never in a game against the house, but the winnings are still governed by the odds, under the luck factor. This principle is related to goals 2, 4, and 5.

h) The module should contain a sub-module dedicated to the interpretations, relativities, psychology, and philosophy of the probability concept, placed at the end of the module. The lessons of this sub-module should be developed by a mathematician assisted by a psychologist and will be a popular presentation of the probability concept in all major views surrounding mathematical probability – classical, inductive, subjective, frequentialist, propensitistic – adapted for the non-mathematician. Students will be walked through the philosophy of probability with no reserve, touching the ontological status of probability and passing through the entire range of interpretations; he or she will be shown the differences between the common-language term and the scientific concept in its various interpretations, the view of probability as both objective and subjective, the difference between *possible* and *probable*, the relationship of probability with the individual psychological degree of belief in the occurrence of an uncertain event. This principle is related to goals 2, 4, and 5.

Besides lectures, the teaching module will have interactive sessions consisting of seminars in which to clarify issues with understanding, solve problems and applications, and perform knowledge tests. As a principle for the interactive portion:

The interactive sessions should also contain discussions on the ongoing gambling experience of the students. These discussions should be focused on the mathematical analysis of the gambling stories, which should be framed within a general model where they are treated as simply one experiment from a series of independent experiments. This principle is related to goals 2, 3, 4, and 5.

The scholastic intervention could have an impact on development of the pathway of gambling in only one type of its processes, namely, the influence of classical and operant conditioning, as this process is common to all models of gambling pathways (Blaszczynski & Nower, 2002); however, the impact could be decisive, since this type of processes corresponds to an early state of the pathway.

While such an intervention will remain optional for gamblers (and future studies can determine the extent to which gamblers will attend an optional intervention), there is still a way of using it as non-optional, namely, as implemented in a proper reduced form in cognitive therapy sessions. Such a direct contribution of mathematics to the psychological intervention can extend beyond providing mathematical information, which is the subject of the next section.

3.3. Further research on the direct contribution of mathematics to psychological interventions

Principle (h), as stated in the previous section is what is missing in the current curricula on either probability theory or mathematics of gambling and I claim it as essential toward an optimal intervention. Such a conceptual component is given the lowest priority, when it is included at all, in the probability courses of secondary and even post-secondary schools, but – paradoxically – could have a role in preventing problem gambling. The conceptual components across all principles stated in the previous section make the transition from an indirect to a direct contribution of mathematics to psychological interventions.

The main subject of further investigation is based on the following premise: The surplus (physical) structure added in reality to the abstract mathematical model of a game (the game in its consumable, commercial, casino form, the environments in which games are running, gambling industry) is that which contains the addiction elements and not *the game itself as mathematical model*. For instance, the *near-miss* effect on slots, obtained through a progressive visualization of a beginning part of a winning combination does not exist in the probabilistic model of that slot game. In that model, only the combinations of stops (holding the symbols) of the reels as elementary events of the probability field do exist, and the expected winning combination has a certain probability. Therefore, there is no *near miss* in the mathematical model, but only the probability of that "near-missed" combination. If the player, through psychological counseling, would reach a state where to have a *representation* of the game only as mathematical model, the *near-miss* effect would vanish and, with it, an important addictive element. A similar observation applies for the *illusion of control*. It is such feature through which gambling addiction is different from other types of addiction (for instance, from smoking, where the addictive elements are in the cigarette itself, which cannot be reduced any more to an essential model).

Then further research is needed for proving theoretically and empirically that an optimal structure and content of a mathematical module following to be implemented into cognitive therapies should contain the *reduction-to-models* conceptual approach, *together* with the *facing-the-odds* component.

Given the interdisciplinary aspects of the processes involved in acquiring the aims of the mathematical intervention, an elaborated research project is needed, which will be the topic of a forthcoming paper. Projects of developing the entire structure and content of both the teaching module and the module for cognitive therapies should also follow the results of this further research.

4. CONCLUSIONS

Past studies on the role of mathematics in problem gambling prevention and treatment have been limited to isolated empirical researches focused on the indirect contribution of mathematics as scholastic intervention. Such empirical researches did not yield conclusive results. Optimizing such mathematical scholastic interventions is possible, by following certain principles that relates to the goals of the intervention. Moreover, given the strong relation of mathematics with the gambling activity, as underlying the games, the potential of mathematics extends beyond the indirect contribution, to a direct one, namely its inclusion into cognitive therapies as a proper mathematical module. Further research is needed for proving theoretically and testing empirically that such a module, basing on the two main principles *facing the odds* and *reduction-to-models*, can improve decisively the cognitive therapies for problem gambling.

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REZUMAT

La întrebarea dacă se poate schimba comportamentul practicanţilor jocurilor de noroc ca rezultat al învăţării matematicii jocurilor de noroc, studiile efectuate până în prezent au oferit răspunsuri contradictorii, nefiind trasă o concluzie clară. În acest articol aduc unele critici cercetărilor empirice care au înclinat să răspundă nu acestei ipoteze, în ceea ce priveşte eşantionarea şi testarea de laborator, şi susţin că o intervenţie scolastică matematică având ca scop prevenţia jocului problematic este posibilă, enunţând principiile care optimizează structura şi conţinutul modulului didactic. Plecând de la aspectele etice ale expunerii faptelor matematice din spatele jocurilor de noroc şi de la cazul jocurilor de sloturi – unde configuraţia parametrică nu este vizibilă – trebuie să tragem linia de demarcaţie între informaţia etică şi cea opţională provenind dintr-o intervenţie scolastică matematică. Susţinând rolul matematicii în prevenţia şi tratamentul jocului problematic, sunt trasate direcţii de cercetare interdisciplinară privind implementarea unui modul matematic adecvat in terapiile cognitive.